King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering
Information and Computer Science Department

ICS 254: Discrete Structures II
Fall Semester 2016-2017 (161)
Final Exam, Wednesday January 18th, 2017.

Name:

ID#:

Instructions:

1. This exam consists of nine pages, including this page, containing **four** questions.
2. You have to answer all **four** questions.
3. The exam is closed book and closed notes. NO CALCULATORS or any helping aides are allowed.
4. The questions are **not** equally weighed.
5. This exam is out of **100** points.
6. You have **120** minutes to finish the exam.
7. Make sure your answers are readable.
8. If there is no space on the front of the page, use the back of the page.

Question Number	Maximum Points	Points
1	20	
2	30	
3	20	
4	30	
Total	**100**	

1) **(20 points):** For each of the following questions, clearly mark the correct answer.
   a) Let a, b and c be integers with $a \neq 0$. If $a|b$ and $a|c$, then
      i) $a|\gcd(b,c)$
      ii) $a|(b-c)$
      iii) Both answers "i)" and "ii)" are correct
      iv) $a|\frac{b}{c}$
      v) $\frac{b}{c}|a$

   b) Which of the following statements is true:
      i) A number k divides the sum of three consecutive integers $n, n+1$, and $n+2$ if and only if it divides the middle integer $n+1$.
      ii) An integer n is divisible by 6 if and only if it is divisible by 3.
      iii) For all integers a, b, and c, $a|bc$ if and only if $a|b$ and $a|c$.
      iv) For all integers a, b, and c, $a|(b+c)$ if and only if $a|b$ and $a|c$.
      v) If m and n are integers, then $m|n$ if and only if $m^2|n^2$.

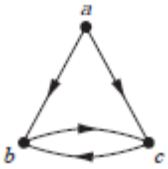
   c) Let L be the least common multiple of 175 and 105. Among all the common divisors $x > 1$ of 175 and 105, let D be the smallest common divisor. Which of the following is correct:
      i) $D = 5$ and $L = 1050$
      ii) $D = 7$ and $L = 1050$
      iii) $D = 3$ and $L = 525$
      iv) $D = 5$ and $L = 525$
      v) $D = 3$ and $L = 35$

   d) To one percent accuracy, the number of integers n in the list $0^4, 1^4, 2^4, \ldots, 1000^4$ such that $n = 1 \pmod{16}$ is
      i) 55%
      ii) 50%
      iii) 45%
      iv) 25%
      v) 6%

   e) The value of $7^{224} \pmod{11}$ is:
      i) 3
      ii) 4
      iii) 5
      iv) 6
      v) 7

   f) Given a relation R, if $R \neq R^{-1}$, where R^{-1} is the inverse relation, then R has to be:
      i) irreflexive
      ii) symmetric
      iii) not symmetric
      iv) asymmetric
      v) antisymmetric

g) The relation that is represented by the graph below is



i) Reflexive, not symmetric and transitive.
ii) Reflexive, antisymmetric and not transitive.
iii) Reflexive, asymmetric and transitive.
iv) Irreflexive, not symmetric and not transitive.
v) Irreflexive, antisymmetric and not transitive.

Questions "h)" to "j)" are on the following partially ordered set (poset):
$(\{\{1\}, \{2\}, \{4\}, \{1, 2\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 3, 4\}, \{2, 3, 4\}\}, \subseteq)$.

h) The set of minimal elements equals
i) $\{\Phi\}$
ii) $\{\{1\}\}$
iii) $\{\{1\},\{2\}\}$
iv) $\{\{1\},\{2\},\{4\}\}$
v) $\{\{1\},\{2\},\{3\},\{4\}\}$

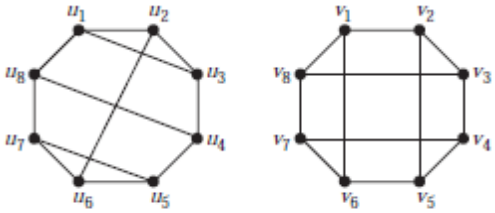
i) The least upper bound of $\{\{2\},\{4\}\}$:
i) is $\{\{2,4\}\}$
ii) is $\{\{2,3,4\}\}$
iii) is $\{\{2\}\}$
iv) is $\{\{4\}\}$
v) does not exist.

j) The set of lower bounds of $\{\{1,3,4\},\{2,3,4\}\}$ is
i) $\{\{3,4\}\}$
ii) $\{\{1,4\},\{2,4\},\{3,4\}\}$
iii) $\{\{4\},\{3,4\}\}$
iv) $\{\{3\},\{4\},\{3,4\}\}$
v) $\{\{1\},\{2\},\{4\},\{3,4\}\}$

2) **(30 points) Graphs:**

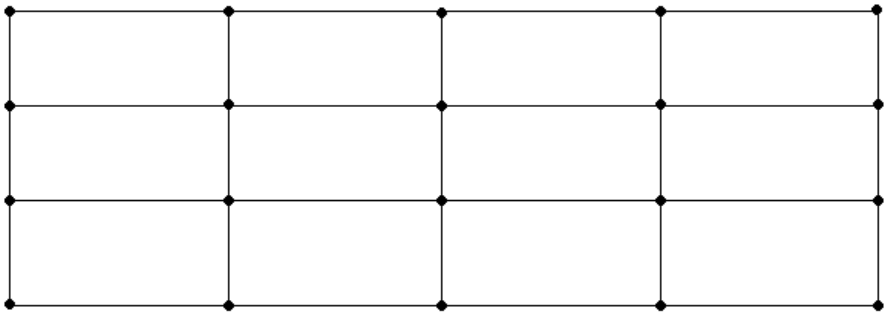
a) (5 points) List <u>all</u> values of m such that the wheel graph W_m is isomorphic to a complete graph K_n. Justify your answer.

b) (5 points) Determine whether these two graphs are isomorphic or not. Justify your answer.



G H

c) (20 points) A grid graph $G(m, n)$ is defined as having m rows and n columns in the form of a grid. For example, $G(4, 5)$ is shown below:



i) Find an expression for the number of vertices, the number of edges and the number of regions in $G(m, n)$.

ii) For which values of m and n is $G(m, n)$ a bipartite graph?

iii) For which values of m and n does $G(m, n)$ have a Hamilton path?

iv) For which values of m and n does $G(m, n)$ have a Hamilton circuit?

v) For which values of m and n does $G(m, n)$ have an Euler path?

vi) For which values of m and n does $G(m, n)$ have an Euler circuit?

3) **(20 points) Trees**:
   a) (6 points) In a full *m*-ary tree T with height h, where all leaf nodes appear at the same level:
      i) Find the number of leaf nodes in T.

      ii) Find the number of internal nodes in T.

   b) (6 points) Consider the following mathematical expression: $\big((x+2) \uparrow 3\big) * \big(y-(3+x)\big) - 5$. Represent this expression using a binary tree.

c) (8 points) For the following two questions, you have to show the order of evaluation for each expression. Evaluating these expressions by first converting them into infix notation will receive 0 points.

i) Evaluate the prefix expression: $+ \; - \; \uparrow \; 3 \; \; 2 \; \uparrow \; \; 2 \; \; 3 \; / \; \; 6 \; \; - \; \; 4 \; \; \; 2$

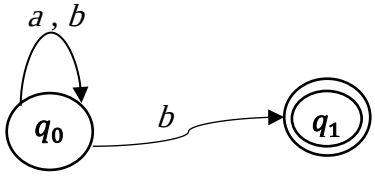
ii) Evaluate the postfix expression: $5 \; \; 2 \; \; 1 \; \; - \; \; - \; \; 3 \; \; 1 \; \; 4 \; + \; \; + \; \; *$

4) **(30 points) Modeling Computation**:
   a) (6 points) Let $\Sigma = \{a, b\}$. Prove that $\{a, b\}^* = a^*.(b.a^*)^*$.

   b) (14 points) Let $\Sigma = \{a, b\}$. Find regular expressions for the following languages over Σ.
      i) $L_s = \{a^m b^n | m \geq 4 \wedge n \leq 3, m, n \in \mathbb{N}\}$.

      ii) $L_t = \overline{L_s}$, the language complement to the one specified in "i)".
      Hint: The regular expression is the union of three expressions!

c) (6 points) Consider the following finite automaton, where q_0 is the initial state.



i) (2 points) Is it deterministic or nondeterministic? Explain.

ii) (4 points) Showing the propagation of states, determine whether the above automaton accepts the string $\underline{abba}$ or not.

d) (4 points) Draw a non-deterministic finite automaton for the regular expression $(ab + ba) * . b$